

Letter to the Editor

Empirical clues about the fulfillment of quantum principles in ecology: Potential meaning and theoretical challenges



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ABSTRACT

A new and wide area of theoretical and methodological overlap between ecology and conventional physics has emerged from the development of an ecological state equation and its consequences. Specifically, the discontinuous (discrete) increase of the ecological equivalent (k_e) of Boltzmann's constant (k_B) suggests a startling hypothesis: most general principles of quantum mechanics could be valid at the ecosystem level. In this paper, we show a single result supported on previous theoretical results as well as on already published data: that a significant and robust straight line adjustment with an intercept at the coordinate's origin between the mean value of eco-kinetic energy per individual and k_e at the inter-taxocenosis scale has a regression constant (slope) whose mantissa coincides with the Planck's constant mantissa at the 1000th level. From this result, we propose two simple equations, with increasing exactness, to assess the expected mean values of individual eco-kinetic energy per survey at the inter-taxocenosis level with a reliable statistical adjustment in comparison with the respective observed values. This result means that the evolutionary process as a whole could be understood as a "staggered propulsion" of a tiny initial clot of life that has been ecologically driven across a discontinuous evolutionary gradient of exchange of information by trophic energy with an increment rate ruled by constant quantum parameters. The potential meaning of this finding for evolutionary ecology and our understanding of the ecosystem functioning is analyzed, and the future challenges to develop a holistic theoretical framework based on this result are stated.

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1. Introduction

The recent proposal of an ecological state equation (ESE; see Eq. (1)) that is structurally equivalent to the ideal gas state equation ($2N\frac{1}{2}m \cdot v^2 = N \cdot k_B/T$; where N : number of molecules; m : molecular mass; v : molecular velocity; k_B : physical Boltzmann constant = $1.3806504 \times 10^{-23}$ J/K/mol; and T : absolute temperature), has two main epistemological consequences: (1) the orthodox point of view against the validity of conventional physics to explain the ecosystem functioning (e.g. Margalef, 1991, pp. 884, 894–895, 904–905; Månsson and McGlade, 1993; Ulanowicz, 2004, p. 341) seems to be an inaccurate reflection of reality. Therefore, the approach to understand the ecosystem functioning based on conventional physics (e.g. Lindeman, 1942; Odum, 1968, 1969; Gallucci, 1973; Jørgensen and Fath, 2004) could have been neglected by mainstream ecological research before producing its most valuable results. (2) A new wide field of research would be opened to derive new knowledge from the theoretical and methodological overlap between ecosystem ecology and several branches of conventional physics.

$$2N_p(E_{ep}) = (N_p \times k_e)/H_p \quad (1)$$

$$2E_{eTp} = (N_p \times k_e)/H_p$$

where

H_p : ecological information assessed as species diversity per plot = $-\sum [(n_i/N) \ln(n_i/N)]$ (Shannon, 1948).

n_i : number of individuals of species i at the plot level.

$$N = \sum n_i.$$

k_e : ecological equivalent of Boltzmann constant = $m_{ep} \cdot I_e^2 \cdot H_p = 1.3806504E\varphi J_e \cdot \text{nat}/\text{individual}$, k_e was empirically assessed for the first time (with $\varphi = 2$ in ruderal vegetation) by Rodríguez et al. (2012, 2013a) starting from the slope trend to 0 in the correlation between $(2N_p\frac{1}{2}m_{ep}I_e^2H_p)/N_p = m_{ep} \cdot I_e^2 \cdot H_p$ and $(1 \cdot I_e^2)/(s_o/m_{eTp})$ in a fully equivalent way to the calculation of k_B . That is to say, k_B was also assessed, more than a century ago, starting from a slope trend to 0 in the correlation between $PV/NT = (2N\frac{1}{2}mv^2)/NT = mv^2/T$ and $((1 \cdot v^2)/(s_f/m_T)) = P$. Where m_T : total physical mass ($N \cdot m$); m_{eT} : total biomass ($N \cdot m_e$) per plot (p), macrostate (m) or survey (s), depending on the studied scale; s_f : physical space occupied by the gas; s_o : mean space per plot or macrostate in the ecosystem; V : gas volume; and P : gas pressure. Rodríguez et al. (2013a) performed additional empirical assessments of k_e (several of these values are included in Appendix A, Table A1, column $k_{e(o)}$), showing that k_e seems to be a general pattern under stationary and quasi-stationary ecological conditions (SEC).

N_p : total number of individuals per plot.

E_{ep} : mean eco-kinetic energy per individual per plot = $\frac{1}{2}m_{ep} \cdot I_e^2$.

E_{eTp} : total eco-kinetic energy per plot = $N_p \cdot E_{ep}$.

m_{ep} : mean individual biomass per plot in kg.

l_e : a dispersal statistical indicator, expressed in δ units, with the appropriate features to replace v (physical velocity) in an analytically invariant way in regard to the replacement of T by H from the ideal gas state equation to Eq. (1), by allowing in such a way the application of physical methods to describe the ecosystem functioning. For additional details about the structure, meaning and calculation of l_e see Appendix A, Section 2; as well as Rodríguez et al. (2012, 2013a,b, 2015).

The publication of additional and complementary results (see Rodríguez et al., 2013a, 2015) indicates that the ESE applies despite body size, taxon, or environment type, because the mantissa of k_e (it indicates the rate in which an individual exchanges information by eco-kinetic energy, and vice versa) of the Boltzmann constant (k_B) is the same for all the stationary ecological assemblages, but φ undergoes discrete variations ($k_e DV$) across a succession of integer values ($-x_i, \dots, 0, \dots, +x_i$). Thus, k_e can be seen as a “universal ecological constant” in a similar fashion as k_B is seen in conventional physics.

This result involves several unexpected aftermaths. One of them is analyzed in this article given that different values of k_e and E_e (sequentially linked to the application of ESE to several kinds of ecological assemblages) are used here as stationary benchmarks to empirically derive clues about the fulfillment of quantum principles in evolutionary biology and ecosystem ecology.

The most recent developments (see Rodríguez et al., 2013b) of this interdisciplinary approach to ecosystem ecology yields two main general results: (1) the old and seemingly unfruitful debate between the competitive exclusion principle (CEP: one species \leftrightarrow one niche: those species that perform the same ecological function cannot coexist; see Hardin, 1960; Darlington, 1972; Gordon, 2000; Wang et al., 2005), and the hypothesis of full functional redundancy (HFFR: several species \leftrightarrow one niche; see Lawton and Brown, 1993; Walker, 1992; Wohl et al., 2004; Petchey and Gaston, 2006; Mayfield and Levine, 2010), would be a function of our poor understanding about the inherent uncertainty in the ecological niche assessment (UEN). (2) The gap between theoretical and empirical ecology in those cases where the diversity of closely related species is too high could be narrowed by introducing a stationary wave model of species coexistence (WMSC, see theoretical explanations and empirical examples in Rodríguez et al., 2013b, pp. 8–11). This model is based on the concurrency between transient equilibrium nodes of CEP, isolated from each other by wide antinodes of limited functional redundancy bounded by the above-mentioned nodes, and vice versa (in a similar way to the standing waves on a string). In such a way, perfect competition would depend on strong inter-specific relationships performed on tiny transient points (nodes) that would act as functional barriers (constraints) between successive dynamic wide areas of coexistence (antinodes) under alternative ecological conditions of partial functional redundancy.

If we take into account ESE, $k_e DV$, UEN and WMSC simultaneously, then the most rational hypothesis, despite its bizarre appearance, is that the ecosystem functioning could be ruled by a set of principles that share some common features with the theoretical framework established by quantum mechanics. As a result, CEP and HFFR can be simultaneously consistent with each other, and this combination could be the best support to reach the highest values of H . This could be the best explanation of ecosystem structure, assuming that a future wave model of the ecosystem functioning based on parameters equivalent to quantum indicators could be theoretically and empirically plausible.

A reliable scientific model, in spite of its very strange nature, is by norm the remaining result after all the obstacles and probable alternative explanations have been removed. Thus, this document

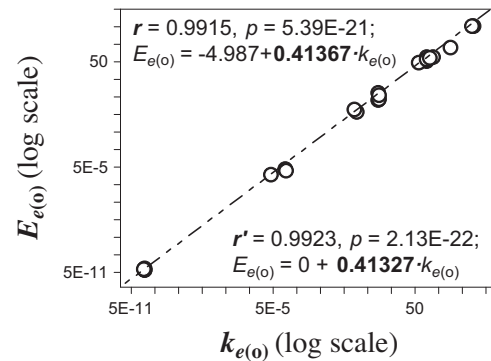


Fig. 1. Pearson's linear correlation between the observed mean value of eco-kinetic energy ($E_{e(o)}$) per individual per survey and the observed mean value of k_e per survey ($k_{e(o)}$). r : correlation with intercept (a) $\neq 0$. r' : correlation with intercept (a) = 0. The logarithmic scale was used to palliate the graphical distortion due to the width of the spectrum of $E_{e(o)}$ values (from $6.7486E-11 J_e$ to $6.0213E+3 J_e$) and $k_{e(o)}$ values (from $1.3807E-10$ to $1.3807E+4$); but correlations were calculated from original values. Number of included surveys = 24 (some points are either clustered or overlapped in the graph due to their similar $k_{e(o)}$ values). Total number of species = 750. Total number of individuals = 88,701,067.

is based on a correlation between the observed mean value of eco-kinetic energy per individual ($E_{e(o)}$) per survey and the respective observed value of the ecological equivalent ($k_{e(o)}$) of Boltzmann constant (k_B), across a wide taxonomic spectrum of ecological assemblages. This single test seems to be consistent enough as to support the probable existence of an ecological equivalent (h_e) of Planck's constant (h). This is the first solid step in order to test the above-mentioned hypothesis. This paper also analyzes the potential ramifications from the existence of h_e in order to improve the reliability of environmental management. Accordingly, the final section of this article explores some factors that are essential to develop a consistent theoretical framework useful to understand several ecological phenomena that cannot be explained by means of conventional classical physics models.

2. Correlation between $E_{e(o)}$ and $k_{e(o)}$

Fig. 1 shows the correlation analysis between the observed mean value of individual eco-kinetic energy ($E_{e(o)} = E_{eTp}/N_p$; see Eq. (1), above) at the survey level and the respective observed (o) value of k_e that was calculated by using the same set of data as well as the same general procedures used by Rodríguez et al. (2013a). Table A1, Appendix A, includes the original set of data used to perform all the calculations in this document; only data from ma1 to css in Table A1 were used to obtain Fig. 1.

One of the values of Planck's constant, depending on the selected units, is (Tipler and Mosca, 2010):

$$h = 4.13566727E - 15 \text{ eV s,}$$

where eV s means “electronvolt \times second”. 1 eV s is the amount of energy ($1 \text{ eV} \approx 1.602176462E-19 \text{ J}$) gained, or lost, per second by a single electron moved across an electric potential difference of 1 V ($1 \text{ eV} \times 4.1356673E-15 \text{ eV s} = 6.62606876E-34 = \text{value of } h \text{ in J} \cdot \text{s}$).

The coefficient of regression (slope, b) of the Pearson product-moment correlation coefficient (r), for $E_{e(o)}$ vs. $k_{e(o)}$ with intercept (a) at $-4.9870 J_e$ in Fig. 1 is $4.13674E-1 J_e$ per individual per $\nabla k_e = 1 J_e$: “eco-Joule”; it is an ad hoc unit that directly results from the calculation of E_{ep} (see Eq. (1), above) taking into account the general fulfillment of the first law of thermodynamics (universal equivalence and free conversion between all the types of energy). Therefore, if the kinetic energy in physics is $E = \frac{1}{2} m \cdot v^2$ (where m : physical mass in kg, and v : velocity in m^2/s^2 ; E is expressed in Joules = $\text{kg} \cdot \text{m}^2/\text{s}^2$), it is completely licit to assume that in ecosystem

ecology $E_e = \frac{1}{2}m_e \cdot I_e^2$ (E_e is expressed in ecoJoules = kg · d²) providing that: (1) the replacing of v by I_e (see calculation of E_{ep} in Eq. (1)) keeps physical and statistical properties invariable in comparison with the classical algorithm (about the features of I_e that have been taken into account to keep this interdisciplinary invariance, see Section 2 in Appendix A), and (2) this assumption must yield results that are simultaneously coherent with physics and ecology from the theoretical and empirical point of view. After all, the primer and main theoretical assumption in order to develop conventional statistical mechanics as a whole was that, if every gas is composed by discrete elements (atoms and molecules) in a permanent motion, then Newtonian mechanics could be useful in order to describe thermodynamics by means of a combination of mechanical and statistical parameters. Eq. (1) is directly based on extending this same assumption to ecology. Rodríguez et al. (2012, 2013a) have provided an overwhelming amount of analytical evidences in favor of item (1). In addition, Rodríguez et al. (2013a,b, 2015) have provided empirical evidences in favor of item (2) by including appropriately-tested results in order to explain several well-known evolutionary and ecological regularities (e.g.: Cope's Rule; Rapoport's Rule; r - K selection theory; equivalence between the ecological succession process and a typical thermal cooling process; correspondence between species diversity gradients under the current socio-economic consumption pressure on ecosystems and Boyle–Mariotte's law; equivalence between the well-known hump-backed curve of trophic energy vs. species diversity and the m_e - I_e trade-off in function of H_p values under SEC; analytical links between wave interference, interspecific relationships and ecological niche assessments in order to explain species coexistence in simultaneous agreement with functional redundancy and the competitive exclusion principle; parallelism between eco-evolutionary principles and physical principles applied to understand the pyramidal structure of trophic flows; etc.) starting from the calculation of E_e in ecology. Thus, the structural similarity between E and E_e seems "to solve the orthodox lack of an equivalent theoretical framework between thermodynamics and conventional ecology" (Rodríguez et al., 2013b, p. 8). Transitively, this general result supports the introduction of J_e as a perfectly acceptable measurement unit in ecology.

The Pearson product-moment correlation coefficient (r') for $E_{e(o)}$ vs. $k_{e(o)}$ with $a=0$ yielded a slope of $b=4.13273E-1 J_e$ (see Fig. 1). The median of these two latter slope values (b from r , and b from r') is $b=4.13473E-1 \approx h'_e = 4.135E-1 J_e$ per individual for each unit of the inter-survey gradient of $\nabla k_e = 1 J_e$ nat/individual.

There is an insignificant statistical difference between the results from r ($p=0.914$ from $t=-0.1097$ for standard error of $a=45.449$) and r' . Therefore, it is acceptable to neglect the role of $a>0$ by assuming $a=0$ in order to arrive to a more general result: the observed mean individual value of eco-kinetic energy ($E_{e(o)}$) per survey should be statistically equivalent to the expected one ($E_{e(e)}$) according to the relationship between two constants; one of them with a general value at the inter-stationary surveys level (h'_e : ecological equivalent of Planck's constant expressed as a factor, derived from SEC data, that multiplies an underlying hidden value, from non-SEC, in J_e ; additional explanations are provided below) and the other one with a particular value at the intra-survey level (k_e) that can be simply interrelated to each other in the following mathematical way:

$$E_{e(e)} = h'_e \times k_e = 4.13566727E-1 J_e \times 1.3806504E\varphi J_e \text{ nat/individual} \quad (2)$$

If this result is correct, then the observed mean value of individual eco-kinetic energy ($E_{e(o)}$, see Eq. (1)) empirically measured for the taxonomic spectrum of surveys included in Fig. 1 as a whole

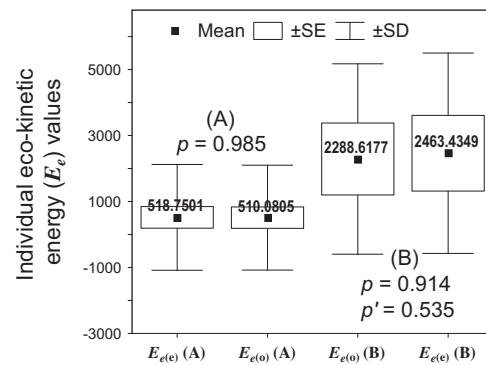


Fig. 2. Comparison between mean values of observed eco-kinetic energy per individual per survey ($E_{e(o)}$), and expected eco-kinetic energy per individual per survey ($E_{e(e)} = h'_e \times k_e$) according to Eq. (2). (A) The comparison based on surveys (ma1 – css in Table A1, Appendix A) included in Fig. 1, and (B) the comparison based on independent surveys (Tsv – css2 in Table A1, Appendix A) not included in Fig. 1. SE: standard error. SD: standard deviation. p = significance level from t -student test. p' = significance level from the non-parametric U test of Mann–Whitney.

should not be significantly different from the expected value ($E_{e(e)}$) according to Eq. (2) (see p value from mean comparison (A) in Fig. 2). It is obvious that, methodologically speaking, the previous comparison (A) could be considered somewhat "tautological" since it has been performed from the same set of data that produced the result generalized by Eq. (2). Thus, a second mean comparison between the same variables ($E_{e(o)}$ and $E_{e(e)}$) using data from 7 additional stationary surveys (their standard stationary testing – according to Rodríguez et al., 2013a – is shown in Table A2, Appendix A) not included in the first set of surveys used to obtain Fig. 1 should coincide with the same result obtained from comparison (A) (see p values from mean comparison (B) in Fig. 2).

In summary, Eq. (2) means that any evolutionary gradient (∇k_e) of trophodynamic conversion between eco-kinetic energy (measured as the mean value of E_e at the survey level) and information (measured as the mean value of species diversity per plot, H_p) under intra-survey SEC implies an inter-taxonomic as well as inter-survey mean increase (or decrease, depending on the net sense – either pro-entropic or anti-entropic – of ecological succession) of individual eco-kinetic energy of $4.13566727E-1 J_e$ per individual per each gradient unit of $1 J_e$ nat/individual between the two border points of the given ∇k_e .

However, Eq. (2) has several drawbacks in spite of its apparent statistical exactness according to Fig. 2 ("As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality", Einstein, 1922, pp. 27–28):

- a) It simulates a large-scale eco-evolutionary process beginning from $E_e = 0$. This is somewhat "strange" since "ex nihilo nihil fit" (nothing comes from nothing). Even in the very unusual realm of quantum mechanics certain level of remnant molecular kinetic energy remains even in the theoretical context in which $T \equiv 0 K \equiv -273.15^\circ C$.
- b) The value of $h'_e = 4.13566727E-1 J_e$ per each $\nabla k_e = 1 J_e$ nat/individual has been obtained by correlation starting from data collected under SEC (only 3 – mif2, rv5 and bA85 – of the 31 surveys included in Table A1 are out of SEC). Under SEC, according to Rodríguez et al. (2013a,b, 2015), there is a biomass-dispersal trade-off in function of diversity values (B-D_{TO-H}: either $m_{ep}^\uparrow \times I_e^{2\downarrow} \times H_p^\uparrow = k_e$, or $m_{ep}^\downarrow \times I_e^{2\uparrow} \times H_p^\downarrow = k_e$; where \uparrow means "increase", and \downarrow means "decrease") that pervades the ecosystem as a whole, acting as an attractor that obstructs any attempt of ecological and evolutionary advancement. As a result, Eq. (2) is a preliminary attempt to assess "eco-evolutionary kinematics" that neglects that in

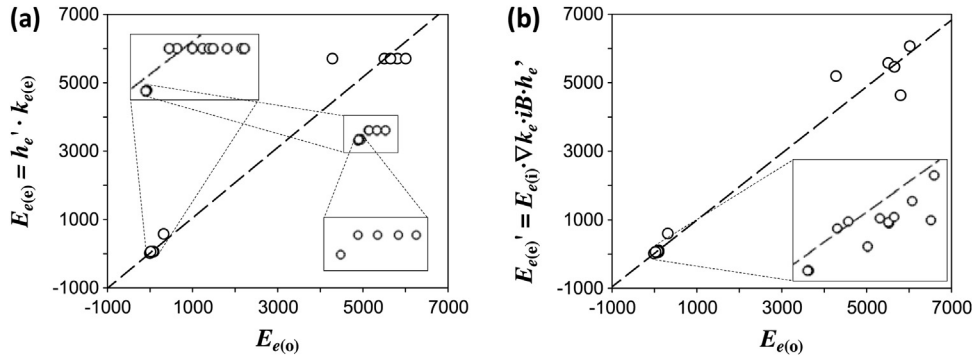


Fig. 3. Correlation results between observed (o) values of eco-kinetic energy (E_e) per survey and expected (e) values of this same parameter calculated by using the ecological equivalents of quantum indicators empirically found in this article. (a) Correlation ($r=0.992$, $p=6.714E-28$) between expected values ($E_{e(e)}$) estimated according to Eq. (2) and $E_{e(o)}$. (b) Correlation ($r=0.991$, $p=8.964E-27$) between expected values ($E'_{e(e)}$) estimated according to Eq. (3) and $E_{e(o)}$.

order to get evolutionary changes between two successive SEC it is necessary to annul the B-D_{TO-H} by getting a non-SEC transition with a significant increase of mean individual biomass and species diversity but without dispersal reduction ($m_{ep} \uparrow \times I_e^{2\uparrow,0} \times H_p \uparrow \neq k_e$, where “0” means neutrality as an alternative option). So, the increase rate of eco-kinetic energy in between two successive SEC should be higher than the h'_e value included in Eq. (2). Figuratively speaking: it is absolutely true that a movie is a kinematic show starting from static frames (f) projected to 24 fps. But it seems to be that, in ecology, there are eight not allowed empty frames by each decimal increment between an allowed frame and the next one (since $k_e = 1.3806504E\varphi$ J_e nat/individual; with $\varphi = -x_i, \dots, -3, -2, -1, 0, +1, +2, +3, \dots, +x_i$). Obviously, if a movie would be projected including only 4.8 frames ($24 \times 2/10$) of each 24, then our knowledge of the movie argument would be quite incomplete.

c) After all, $h=4.13566727E-15$ eV·s is a proportion calculated under acceleration conditions (electric potential difference) that connects two other physical units (J and s) whose existence was originally assessed from the stationary exchange of energy between cavity radiation and cavity walls (see BBR, below). However, an ecological mantissa equivalent to h (6.62606876) does not directly arise from Eq. (2).

d) As a result of items (b) and (c), Eq. (2) generates an excessively staggered assessment of eco-kinetic energy values (see Fig. 3a). That is to say, all the expected values of E_e for a given set of similar k_e values are equals. But the observed values of E_e are not so staggered in regard to observed k_e values that belong to a common shared ecological context (either equal or trophodynamically equivalent ecological assemblages). Besides, from item (b), the empirical source of Eq. (2) leis in an exploration of SECs, but the ecological context that we want to forecast is transitional and it overcomes the influence of B-D_{TO-H} (non-SEC, see item (b)). This indicates toward the probable existence of some kind of “hidden variable” (using a term first introduced by Bell, 1966; see introduction of iB in Eq. (3)) during inter-SEC transitions that needs to be added to Eq. (2) to obtain a better estimation of $E_{e(o)}$. In other words, it is very probable that Eq. (2) underestimates $E_{e(o)}$'s values because h'_e has been derived using only observations from SEC to SEC, with very scarce values in between. We are dealing here with the very frequent problem (see, e.g.: Schumpeter, 1982, p. 1051) of estimating dynamic transitions from a sequence of static observations.

In summary from items (a)–(d), it would be appropriate to introduce certain modifications on Eq. (2) in order to overcome these drawbacks to get a better fitting between the expected and observed values of E_e . For example (from data in Table A1, Appendix

A), it is possible to compare the statistical behavior of the expected values of eco-kinetic energy ($E_{e(e)}$) per survey obtained from Eq. (2) with alternative expected values ($E'_{e(e)}$) obtained from combining (1) quantum indicators linked to Eq. (2); (2) the expected value of k_e , and (3) the mean observed value of E_e in the lowest limit of taxonomic gradient (e.g.: the four surveys of marine microalgae labeled as “ma” in Table A1) by mean of the following equation (variables highlighted in italics and constants highlighted in boldface):

$$E'_{e(e)} = E_{e(i)} \times \nabla k_e \times (iB \cdot h'_e) \quad (3)$$

$$E'_{e(e)} = E_{e(i)} \times \nabla k_e \times h_e$$

where:

$E_{e(i)}$: evolutionarily initial eco-kinetic energy (i.e. that observed value of E_e in microscopic individuals given the general fulfillment of Cope's rule in the largest evolutionary scale; see Cope, 1896; Kingsolver and Pfennig, 2004; Hone et al., 2005; Rodríguez et al., 2013a) for ∇k_e in Table A1 as a whole. That is to say, $E_{e(i)}$ is the mean value of eco-kinetic energy per individual per plot from the original data in surveys ma1, ma2, ma3, and ma4 (165 plots), taken as a whole = $7.86254E-11$ J_e per individual.

∇k_e : $k_{e(o),(j \geq i)} / k_{e(e),(i)}$; where $k_{e(e),(i)}$ is the theoretically expected value of k_e that coincides with $E_{e(i)}$ in Table A1, and $k_{e(o),(j \geq i)}$ is the respective observed value of k_e for a given survey (j) with an average value of eco-kinetic energy per individual higher than or approximately equal to $E_{e(i)}$.

iB : 1 inboltz; i.e., the presumptive ecological equivalent of 1 eV (electronvolt, see above) that denotes the average amount (per individual per taxocenosis) of eco-kinetic energy (E_e in J_e) gained, or lost, by a single individual “moved” (either from the evolutionary or ecological point of view) across a trophodynamic potential difference of 1 ecoBoltz ($\nabla k_e = 1$ J_e nat/individual) along a non-steady transition between two allowed extreme successive values of the ecological equivalent of Boltzmann constant (k_e) estimated under SEC. $1 iB = 1.602176462E-19 \times 1E+19 = 1.602176462$.

h'_e : $4.13566727E-1$.

$h_e = iB \cdot h'_e = 6.6260727E - 1$ J_e (J_e nat/individual).

Eq. (3) (data and results in Table A3, Appendix A) means, in an “earthly” language, that starting from the microscopic root of the evolutionary tree there has been an initial “clot of life” ($E_{e(i)}$) that has been “ecologically driven” across a step by step evolutionary gradient (∇k_e) of exchange of information by eco-kinetic energy at a constant eco-kinetic energy increment rate of h_e per each value of $\nabla k_e = 1$ J_e nat/individual.

The title of this article plainly states that it is only an empirical approach. Therefore, the presumptive validity of Eq. (3) would depend on reaching a better fitting between expected and observed

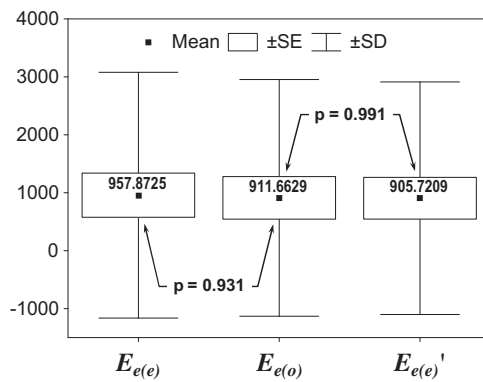


Fig. 4. Comparison between mean values of observed eco-kinetic energy per individual per survey ($E_{e(o)}$), expected eco-kinetic energy per individual per survey ($E_{e(e)}$) according to Eq. (2), and expected eco-kinetic energy per individual per survey ($E_{e(e)'}^1$) according to Eq. (3) for the set of surveys included in Table A1 (Appendix A) as a whole. SE: standard error. SD: standard deviation. p = significance level from t -student test.

values of E_e than Eq. (2), by solving the drawbacks previously commented in items (a)–(d), by means of the structural modifications from Eq. (2) to Eq. (3).

Eq. (3) provides a non-staggered correlation (in regard to each realm of comparable k_e values) of the observed values ($E_{e(o)}$) of eco-kinetic energy per survey (see Fig. 3b in comparison with Fig. 3a). Besides, Eq. (3) provides a better estimation of the mean value of $E_{e(o)}$ than those obtained from Eq. (2) (see Fig. 4). That is to say, the meaning of Eq. (3) commented in the previous paragraph seems to be consistent with reality.

3. Potential meanings for ecology and environmental management

In conventional quantum mechanics (see Halliday et al., 1999; Tipler and Mosca, 2010), only discrete variations of energy (ΔE), between two different stationary states of a given particle, are allowed. Therefore, ΔE is an integer multiple of an elemental minimum amount of energy or “quantum of action” whose value is precisely h . This means that atomic oscillators do not exchange an arbitrary amount of ΔE but only “energy packages” composed by a set of indivisible energy units defined by:

$$\Delta E = \left(n + \frac{1}{2}\right) \times h \times f_0, \quad n = 1, 2, 3, \dots \quad (4)$$

where f_0 is the lowest or *fundamental oscillation frequency* associated to the longest wavelength (λ_0) of the first harmonic (i.e.: a single oscillation with 1 point of maximum–minimum in between the two steady extreme points of a string).

In such a way, physical energy is quantized or “discretely exchanged” starting from a basic or elemental amount (h). The product $h \cdot f$ defines the variation of energy (ΔE) allowed between two contiguous energy levels separated to each other by a gradient, just as two contiguous stationary ecological states are evolutionary separated by different k_e values (see ∇k_e in Eq. (3)).

The first theoretical emergence of the quantum of action took place in regard to the analysis of black body radiation (BBR, light of low frequency emitted by hot bodies) by Max Planck. He found that the only practical way of obtaining a reliable statistical distribution of E densities (expressed as spectral radiance) in function of f values to describe the BBR was the *arbitrary introduction* of an “artificial” constant (h) in regard to the instant in which the atomic oscillators exchange energy with the black body cavity. Einstein (1905) showed that the *empirical proposal* from Planck was also valid for isolated photons whose energy value is $E = h \cdot f = h \cdot (c/\lambda)$;

where f : frequency of light, c : speed of light, and λ : wavelength of light.

Compton (1923) derived the mathematical relationship between the shift in wavelength and the scattering angle of the X-rays by assuming that each scattered X-ray photon interacted with only one electron. His paper concludes by reporting on the experiment that verifies the mathematical relation: $\lambda' - \lambda = [h/(m_e \cdot c)] \cdot [1 - \cos \theta]$, where λ' : initial wavelength of the X-ray photon; λ : wavelength after scattering; m_e : electron rest mass, and θ : the scattering angle. This experiment showed the *wave-particle duality* of light.

Finally, de Broglie (1924) proposed the existence of “matter waves”, by extending the above-mentioned duality from light to particles for showing that every particle of matter (e.g. electrons, neutrons, protons, etc.) with a rest mass m and a velocity v is associated to a real “pilot wave” (the first physical example of a “hidden variable”) whose λ is related to the momentum of the particle ($p = m \cdot v$) by the equation:

$$\lambda = \frac{h}{p} = \frac{h}{m \times v} \quad \therefore \quad h = \lambda \times m \times v \quad (5)$$

That is to say, the quantum nature of matter, at the sub-atomic scale, could have been initially discovered either by means of experiments with photons ($m=0$) or by means of experiments with particles in which $m > 0$, since quantum principles are equally valid for both. If quantum mechanics has its roots in experiments with electromagnetic waves it is only because experiments with photons are technically easier as well as more conventional (i.e. in agreement with the classic double-slit experiment performed with photons by Thomas Young in 1803) than those experiments performed with particles with $m > 0$.

Now, according to Figs. 1 and 4, we are in front of the astonishing possibility that the previously sketched theoretical development could also be applicable to ecology. The potential implications of this result are very difficult to forecast since the validity of quantum mechanics has been conventionally restricted to the subatomic world, with very important but limited macroscopic effects (e.g. superfluidity of hydrogen, electrical superconductivity, Meissner effect or “expulsion” of the magnetic field, etc.).

Nevertheless, we can attempt to explore some possibilities from the existence of h_e for evolutionary theory and ecosystem ecology. For example, everybody in this field suppose, since the time of Charles Darwin, that species diversification implies a “compartmentalization of limited resources” in a process in which a very wide initial ecological niche is evolutionarily subdivided to “accommodate” more and more species (adaptive radiation, AR). In fact, the compartmentalization of a given spectrum of H_p values by means of a succession of nodes of CEP and antinodes of HFFR (see above, as well as Rodríguez et al., 2013b) in order to promote species coexistence it is only a very simple model to put AR in physical terms.

However, it is evident that such a subdivision of resources *cannot go into infinite*. It should have a *limit* beyond which such a compartmentalization is unfruitful, and to divide the “ecological pay” regarding a constant value of trophic efficiency (constant k_e value) is not useful anymore (ecological saturation, e.g. Schippers et al., 2001; Roelke and Eldridge, 2008; Narwani et al., 2009). Then it is necessary “to expand the ecological pay” by jumping up from the evolutionary point of view (∇k_e) toward higher values of biomass and eco-kinetic energy per individual and lower values of energy availability (see Rodríguez et al., 2013a,b). If the result derived from Figs. 1 to 4 is a real ecological alternative, then this *limit* can be given by the evolutionary influence of h_e . That is to say, the quantum nature of the exchange of eco-kinetic energy would not allow the partition of energy gradients below the value of h_e . In

such a way, the reduction of the “distance” (antinodes) between successive nodes in combination with their respective values of biomass and dispersal would have an insurmountable value to avoid the collapse of the system due to a massive coalescence of CEP nodes that would produce a sudden mass extinction of species (i.e. the ecological equivalent of the ultraviolet catastrophe in regard to BBR, theoretically resolved in physics by the introduction of h).

What would happen if h_e , unlike k_e , is a general constant without a hierarchical variation? In that case, the general fulfillment of the presumptive ecological relationship (starting from Eq. (5))

$$h_e = \lambda_e \times p_e = \lambda_e \times m_{ep} \times I_e \quad (6)$$

(where λ_e is ecological wavelength, whatever the nature of supposed ecological waves) would imply a gradient from evolutionarily early microbial communities of larger λ_e and lower p_e (since the long sequence of evolutionary changes on the Earth has been characterized by the increase of p_e mainly due to the increase of biomass – m_{ep} – with the evolutionary span –Cope’s rule) to evolutionarily late macro-fauna communities of shorter λ_e and higher p_e to keep the validity of Eq. (6).

A decreasing λ_e is equivalent to reduce the distance between nodes of CEP (reduction of the antinodes of HFFR that promote species coexistence under similar ecological conditions at the small scale), and vice versa. Then Eq. (6) would explain, on the one edge of the gradient, *the paradox of the plankton* (tiny organisms of very low p_e and large λ_e that are able to support a higher intensity of species coexistence per space unit; see Hutchinson, 1961; Wilson, 1990; Schippers et al., 2001; Scheffer et al., 2003; Roy and Chattopadhyay, 2007). Eq. (6), on the opposite edge of the gradient, would also explain *the higher risk of extinction of large body size animals*, with a higher p_e value and a shorter λ_e that are unable to support a higher intensity of species coexistence per unit of space under equivalent local conditions, in comparison with the opposite edge of the taxonomic gradient.

After all, if life has an essentially wave nature, then our socio-economic interaction with short-wavelength ecological systems (impact-intolerant macro-fauna communities of very high biodiversity) should produce a greater disturbing influence than our interaction with large-wavelength ecological systems (i.e. impact-tolerant microbial communities, mainly if their H values are low). This would be a similar case to that of microscopic observation: an electronic microscope is able to produce sharper and more magnified images than an optical one just because in the first case a shorter wavelength is used to observe a given object. In an equivalent manner, short-wavelength ecological systems would be more capable to magnify the human impact on the environment than large-wavelength ecological systems.

The most probable option is that nature has been able to palliate the above-mentioned problem by means of a variable value of h_e in the intra-survey context depending on the predominant evolutionary stage, in a similar way to the case of k_e . But even under this circumstance, as commented above, there should be an intra-survey quantum limit for niche compartmentalization. Therefore, the influence of the respective value of h_e should have an outstanding role in ecology and evolutionary biology. After all, the very small value of the physical Planck’s constant (h) is the main theoretical obstacle for applying conventional quantum principles to macroscopic complex systems (Haven, 2008, offers a review of quite non-conventional theoretical attempts, based on complex mathematical assumptions, for applying quantum considerations to certain type of macroscopic systems), because with an h magnitude of the order of either E–15 (in eVs) or E–34 (in Js), large mass bodies would have a negligible λ value according to Eq. (5). But this barrier is precisely the first obstacle that has been removed by obtaining an alternative value of h_e in this article.

In a higher and more general analytical level, the result from Figs. 1 to 4 poses a deeper question: What are the analytical limits of physics? Are we unknowingly living in a fully quantum world? Perhaps the key point in this regard is the definition of measurement, stated by Niels Bohr in quantum mechanics, as *an interaction between instruments that belong to two different scales of the physical world in order to obtain information*. It is clear that getting information is the ultimate net result of our interaction with nature but, why do we should assume that we do not belong to the same physical scale of nature in order to admit that quantum ecological properties emerge from our “measurements” on natural environment? An event that sharply delimits a scale difference between nature and society is not very clear either in genetics (it has been impossible to find a particular ancestral single mutation that could be responsible for the cascade of transformations from a primitive primate to a human being) or in economics (human impact on nature seems to be gradual, the higher the economic power, the stronger the impact) or in paleontology (e.g. there is almost a continuum in the ability for using “instruments” in the border of transition between nature and society).

However, there is an event that sharply delimits a scale difference between nature and society in ecology: if we equate the meaning of the diversity of ecological niches in natural ecosystems with the meaning of the diversity of productive socio-economic functions in society (see e.g. McLaughlin, 1930; Clark et al., 1964; Parr, 1965, p. 22; Marcuzzi and Camuffo, 1968; Nourse, 1968, p. 176; Garrison and Paulson, 1973; Hackbart and Anderson, 1975; Odum, 1980, p. 166; Attaran, 1986; Kort, 1988; Simon, 1988; Margalef, 1991, p. 870; Rodríguez and González, 2000; Matutinović, 2001; Mizutani et al., 2003; unfortunately, all of them, despite their abundance, with a null effect on the orthodox economic paradigm) then we are able to grasp that *we belong to the only species that has broken the boundaries imposed by the principle of competitive exclusion* (i.e. in nature one species \leftrightarrow one ecological function or niche; but in society one species \leftrightarrow a potentially unlimited number of socio-economic functions). This is the essential reason of inter-scale differences in our interaction with nature in order to get information. Thus, this is **the main foundation** for expecting the arising of quantum effects when we study the ecosystem functioning by applying physical principles.

A full translation of principles and methods from quantum mechanics to ecology would mean that one of the speculations proposed by Thorne in regard to the forecasted achievements of physics in the 21st century (“... for the first time in human history, we will watch human-sized objects behave quantum mechanically”; Thorne, 2003, p. 92) would become true but, paradoxically, in the realm of biology instead of in the realm of physics itself. After all, even the conventional ecosystem ecology can be understood in the last instance (according to some classical authors as Margalef, 1963; Odum, 1968, 1969, 1980), as a set of emergent properties starting from physics. So the essential traits of the ecosystem could be backward-assembled until reaching their primary contact points with physics.

In quantum mechanics, two particles very far away to each other that were interrelated by a previous interaction or a common shared origin will remain connected in the future. As a consequence, any measurement performed on one of them has an instantaneous effect that changes the properties of its correlative particle (quantum entanglement; see Einstein et al., 1935; Schrödinger and Dirac, 1936; Aspect et al., 1982; Salart et al., 2008; Yin et al., 2013). All the species on the Earth have a common shared evolutionary origin approximately 3.7 billion years ago (Ohtomo et al., 2014) or maybe as early as 4.252 billion years ago (Nemchin et al., 2008). That our environmental impact, even on a single species, has a diffuse effect on the rest of life in all the surface of the Earth is an old and suggestive conjecture in ecology. Nevertheless, it

is hard to prove this idea in a reliable and quantitative way by using current models. But by using a future quantum ecological model of the ecosystem functioning perhaps it would be possible to quantify some sort of “ecological entanglement”, which would have an insurmountable value to assess the magnitude and diffusion of our influence on the natural environment.

On the one hand, the previous exploration of future possibilities does not obey any kind of conservative thinking since science does not advance thanks to this kind of thought; on the contrary, it needs fresh and audacious ideas useful to unveil the mysteries of life (“*The most beautiful and deepest experience a man can have is the sense of the mysterious – . . . – He who never had this experience seems to me, if not dead, then at least blind*”; Einstein, 1932, p. 262). But on the other hand, we need to be aware about the theoretical and empirical obstacles associated to further developments of this result (following section).

4. Future challenges

The method applied to obtain h_e in Fig. 1 and Eq. (3) is quite unconventional in comparison with physics because there is not any sort of gradient of k_B values to obtain empirical evidence in favor of the existence of h in conventional physics. At the first glance, this has a positive side: in contrast to physics, h_e emerges in ecology as a natural result from the application of a classical procedure (ESE, see Eq. (1)) instead of as a product of a contradiction between classical mechanics and observations, as it occurred with h regarding the description of BBR. However, at the second sight, this unconventionality implies that we will need an overwhelming amount of additional evidence in favor of h_e to promote a reliable quantum model of the ecosystem. Coherently, a complete and consistent conceptual extrapolation from quantum mechanics to ecology by keeping the interdisciplinary coherence between both sciences would also be necessary.

Besides, it is relatively easy to obtain a clear result in this case starting from a strong gradient at the inter-survey and inter-taxonomic level (Fig. 1) that is more relevant for evolutionary biology than for ecology. Contrastingly, one of the main future challenges is to perform an independent (i.e. without using an auxiliary constant as k_e) and statistically significant assessment of h_e values at the intra-survey level under SEC, where the internal gradients are substantially weaker and more relevant for our understanding of the most frequent status of ecosystem functioning.

The latter challenge requires finding a reliable set of variables useful to glimpse the real nature of trophodynamic waves in coherence with several well-known and undoubtedly proven principles of conventional ecology. This would convert the main result of this article from an empirical to a theoretical one. According to Rodríguez et al. (2013b) a couple of abundance waves that intertwines two dominant species to each other could be used for such a goal. But it seems that we would need a more inclusive variable than abundance itself because of two main factors: (1) the previously mentioned method would only embrace a tiny fraction of the ecosystem as a whole; (2) the empirical exploration of this option by Rodríguez et al. (2013b, p. 9, Fig. 5d) indicates that even a single pair of species can be performing several kinds of either stationary or non-stationary inter-specific relationships depending on local ecological conditions. This does not facilitate the obtaining of consistent intra-survey results about h_e .

Even if the previously sketched obstacles are overcome, then we will need to find a second set of variables to prove the influence of Heisenberg's indetermination principle at the ecological scale, as a fundamental requisite to assess ecological entanglement in practice. Then, swimming against the generalized academic current in favor of intellectual hyper-specialization and interdisciplinary

isolation that pervades contemporary science, a huge interdisciplinary effort should be performed to endow this new theoretical framework with a comprehensive sense for ecologists, as well as for physicists, in order to improve our understanding about nature–society interactions and development sustainability.

In summary, regardless the suggestive nature of the result from this article, it is absolutely sure that this is only a drop of initial information into an ocean of uncertainty; a sort of first step in a long road whose end is impossible to forecast. However, despite all these challenges, as in physics, this shy first step is the most essential beginning if the above mentioned hidden field for ecological research truly exists.

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Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.ecolmodel.2015.01.023>.

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